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MathsMethods.com.au Exam Solutions

These are only suggested solutions and are in no way endorsed by the VCAA. There are multiple ways of tacking each problem and this booklet generally only outlines one of them.

STUDENT NUMBER

MATHEMATICAL METHODS

2017 Exam 2 (Multiple Choice) Solutions

The average mark for the multiple-choice section was 59%

QUESTION AND ANSWER BOOK

Structure of book

Section	*Number of questions*	*Number of questions to be answered*	*Number of marks*
A	20	20	20

There are also clickable links to more free resources.

Some of the questions have additional resources for you to access for free, just click the link.

MathsMethods.com.au is dedicated to help every Methods student and we have plenty of free and paid resources that we have put our heart and soul into – but I want to know what you think!

Email any feedback or suggestions on: alex@mathsmethods.com.au

I hope you find this helpful.

Alex Bell – Creator of MathsMethods.com.au

SUMMARY OF THE SOLUTIONS

SECTION A - Multiple Choice (Each question is worth 1 mark)

1	2	3	4	5	6	7	8	9	10
C	D	C	E	A	C	B	A	A	D

11	12	13	14	15	16	17	18	19	20
D	C	E	D	B	C	D	D	D	B

SECTION A - Multiple Choice (Each question is worth 1 mark)

Question 1 - Period and range of $f(x) = \sin(2x) - 1$

Click here for **Maths**Methods.com.au resource: *Sin, Cos and Tan Graphs*

Period of any sin or cos graph is $\frac{2\pi}{k}$ where k is any value in front of x

Period is $\frac{2\pi}{k} = \frac{2\pi}{2} = \pi$ → It can only be **A** or **C**

Range means the y values of the functions.
In this case it'll be the amplitude plus vertical translations

Amplitude $= 5$, ($y = 5\sin(x)$ would have a range of -5 to 5)

Vertical translation is -1, so this affects the range

Range is -6 to 4

The answer is C. period is π and range is $[-6, 4]$ ANSWER

Question 2 – x values where $f'(x)$ is negative

Click here for **Maths**Methods.com.au resource: *Calculus Basics*

The word "Interval" implies they want x values.

$f'(x)$ is gradient, so the question asks for negative gradient.

The gradient is negative for $\mathbf{-3 > x > \frac{5}{3}}$ this can be written as $\left(\mathbf{-3, \frac{5}{3}}\right)$

The answer is D. $\left(-3, \frac{5}{3}\right)$ ANSWER

Question 3 – Probability of different colours when selecting 2 marbles

Click here for **Maths**Methods.com.au resource: *Probability*

5 Red marbles 3 Yellow marbles total of 8 marbles - **No replacement**

To different colours, it would be R then Y or Y then R

$\Pr(RY) = \frac{5}{8} \times \frac{3}{7} = \frac{15}{56}$ $\Pr(YR) = \frac{3}{8} \times \frac{5}{7} = \frac{15}{56}$

We add these together

$\Pr(\text{Different colours}) = \Pr(RY) + \Pr(YR) = \frac{30}{56} = \frac{15}{28}$

The answer is C. $\frac{15}{28}$ ANSWER

Question 4 – Find value of $f\left(g(3)\right)$ when $g(3) = 2$ and $f(2) = 5$

Click here for **Maths**Methods.com.au resource: *Composite Functions*

$g(3) = 2$ and $f(2) = 5$ $f\left(g(3)\right) = f(2) = 5$

The answer is E. $f\left(g(3)\right) = f(2) = 5$ ANSWER

Question 5 – Sample proportion from 95% confidence interval from large sample is (0.039, 0.121)

Click here for **Maths**Methods.com.au resource: *The Basics of Stats*

95% confidence interval is $(0.039, 0.121)$

95% confidence interval formula is $\left(\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}, \hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}\right)$

Where $\hat{p}$ is sample proportion

$\hat{p} - 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.039$ Equation 1

$\hat{p} + 1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 0.121$ Equation 2

Add these two together to cancel out the $1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ Equation 1 + Equation 2 =

$2\hat{p} = 0.160$ (notice that the $1.96\sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$ cancel out)

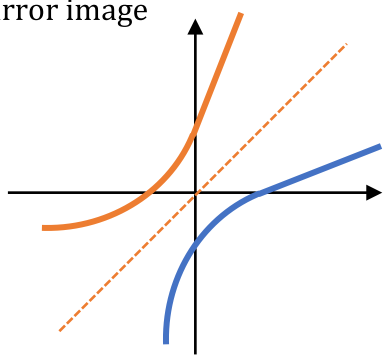
$\hat{p} = 0.080$

The answer is A. $\hat{p} = 0.080$	ANSWER

Question 6 – Find the graph of the inverse

Click here for **Maths**Methods.com.au resource: *Inverse Functions*

The inverse is reflected in $y = x$, so sketch one in and then draw a mirror image



The answer is C.	ANSWER

Question 7 – Find no solution for $(p-1)x^2 + 4x = 5 - p$

Click here for **Maths**Methods.com.au resource: *Parabolas*

A root is a solution for an equation (usually the value of x).

quadratic equation $= \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ when $a^2 + bx + c = 0$

$\boldsymbol{b^2 - 4ac} < \mathbf{0}$ means no real solutions, because you can't have a negative root.

$(p-1)x^2 + 4x - (5-p) = 0$

$\boldsymbol{b^2 - 4ac} = 4^2 - 4 \times (p-1) \times -(5-p) = -4p^2 + 24p - 4$ (using the calcualtor)

$\mathbf{-4p^2 + 24p - 4 < 0}$

This isn't any of the answers – all the answers have $\boldsymbol{p^2}$ rather than $\boldsymbol{-4p^2}$

Let's divide both sides by 4

$\mathbf{-4p^2 + 24p - 4 < 0}$

$\mathbf{-p^2 + 6p - 1 < 0}$

Let's divide both sides by -1

$\mathbf{-p^2 + 6p - 1 < 0}$

$\mathbf{p^2 - 6p + 1 > 0}$

Notice that the $<$ turned into a $>$, this happens whenever a negative is multipled.
For example, $5 < 10$ and $-5 > -10$

The answer is B. $p^2 - 6p + 1 > 0$ ANSWER

Question 8 – Solve $y = a^{b-4x} + 2$ for x, given a is larger than 0

Click here for **Maths**Methods.com.au resource: *Understanding Logarithms*

Your calculator will not give you a suitable solution as it only gives answers in $\log_e$

$m = a^n$ then $n = \log_a(m)$

$y = a^{b-4x} + 2$

$y - 2 = a^{b-4x}$ then $b - 4x = \log_a(y-2)$

$-4x = \log_a(y-2) - b$

$x = -\frac{1}{4}(\log_a(y-2) - b)$

This isn't any of the answers.
All the answers are positive so, let's put that negative inside the bracket.

$x = -\frac{1}{4}(\log_a(y-2) - b)$

$x = \frac{1}{4}(-\log_a(y-2) + b)$

$x = \frac{1}{4}(b - \log_a(y-2))$

The answer is A. $\frac{1}{4}(b - \log_a(y-2))$ ANSWER

Question 9 – Average rate of change of $f(x) = x^2 - 2x$ is 8 over the interval $[1, a]$, find the value of a

$$\text{average rate of change} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(x_2) - f(x_1)}{x_2 - x_1} = 8$$

$$\frac{f(a) - f(1)}{a - 1} = 8$$

This can be done in the calculator and there is also probably a "rate of change" function you can use as well.

$$solve\left(\frac{f(a) - f(1)}{a - 1} = 8, a\right) \quad \rightarrow \quad a = 9$$

The answer is A. $a = 9$	ANSWER

Question 10 – What does $\begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ transform $y = 3\sin\left(2\left(x+\frac{\pi}{4}\right)\right)$ into?

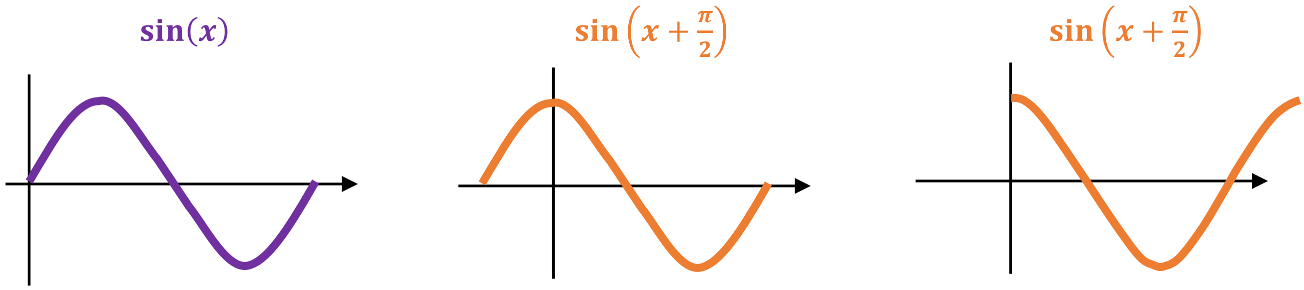
Click here for **Maths**Methods.com.au resource: *Transformations*

Map means to transform a function. This is how a matrix affects a function:

$$\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow y = bf\left(\frac{1}{a}x\right) \qquad \begin{bmatrix} 2 & 0 \\ 0 & \frac{1}{3} \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} \rightarrow y = \frac{1}{3} \times f\left(\frac{1}{2}x\right)$$

$$y = 3\sin\left(2\left(x+\frac{\pi}{4}\right)\right) = y = \frac{1}{3} \times 3\sin\left(2\left(\frac{1}{2}x+\frac{\pi}{4}\right)\right) = \sin\left(x+\frac{\pi}{2}\right)$$

This isn't any of the options. So, let's sketch it.



$\sin\left(x+\frac{\pi}{2}\right) = \cos(x)$

You can could check each answer (A, B, C, D and E) to see which $= \sin\left(x+\frac{\pi}{2}\right)$

It would go like this:

$\sin\left(x+\frac{\pi}{2}\right) = \sin(x+\pi)$ FALSE

$\sin\left(x+\frac{\pi}{2}\right) = \sin\left(x-\frac{\pi}{2}\right)$ FALSE

$\sin\left(x+\frac{\pi}{2}\right) = \cos(x+\pi)$ FALSE

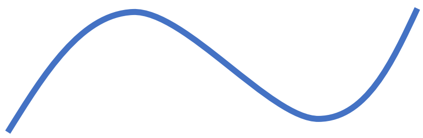
$\sin\left(x+\frac{\pi}{2}\right) = \cos(x)$ TRUE!

The answer is D. $y = \cos(x)$ ANSWER

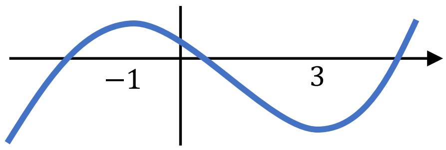
Question 11 – $y = x^3 + ax^2 + bx$ has local max at x = -1 and min at x = 3

Click here for **Maths**Methods.com.au resource: *Stationary Points*

First off, let's have a look at what we're dealing with. It's a positive cubic (highest power is 3 and x^3 is positive) with two turning points, so we know it will be this shape:



We also know that the maximum is at $x = -1$ and minimum is a $x = 3$



These stationary points occur when $f'(x) = 0$.

$f(x) = x^3 + ax^2 + bx$
$f'(x) = 3x^2 + 2ax + b$
Hence, $f'(-1) = 0$ and $f'(3) = 0$

$f'(-1) = 3(-1)^2 + 2a(-1) + b = 3 - 2a + b = 0$
$f'(3) = 3(3)^2 + 2a(3) + b = 27 + 6a + b = 0$

Use your calculator to solve these equations.
You could also use your calculator to do all the above steps as well.

$a = -3$ and $b = -9$

The answer is D. $a = -3$ and $b = -9$ ANSWER

Question 12 – $\sin(2x) = \frac{\sqrt{3}}{2}$ over $[-\pi, d]$, where adding all solutions is $-\pi$.

Click here for **Maths**Methods.com.au resource: *Sin, Cos and Tan Basics*

$$solve\left(\sin(2x) = \frac{\sqrt{3}}{2}, x\right) | -\pi < x < 2\pi$$

Note, make sure your calculator is in radians.

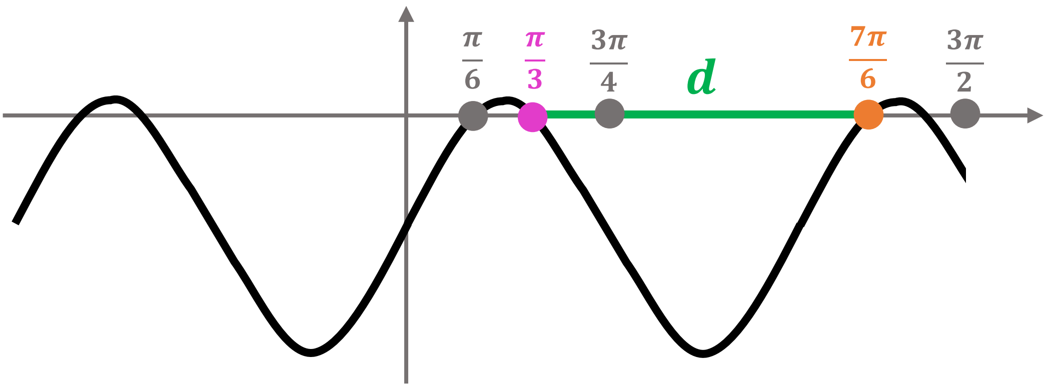
I picked 2π just to get a bunch of solutions. I could have picked a larger or smaller number.

$$x = -\frac{5\pi}{6}, -\frac{2\pi}{3}, \frac{\pi}{6}, \frac{\pi}{3}, \frac{7\pi}{6}, \frac{4\pi}{3}$$

Now just start adding them up until you get to $-\pi$

$$-\frac{5\pi}{6} + -\frac{2\pi}{3} + \frac{\pi}{6} + \frac{\pi}{3} = -\pi$$

Therefore, the domain must be larger than $\frac{\pi}{3}$ but smaller than $\frac{7\pi}{6}$.
Let's look at all the possibilities.



If you're not good with fractions, turn the answers into decimals on your calculator and see which answers are between 1.0472 to 3.66519 (which are the decimal version of $\frac{\pi}{3}$ and $\frac{7\pi}{6}$).

The only answer is $\frac{3\pi}{4}$.

The answer is C. $d = \frac{3\pi}{4}$	ANSWER

Question 13 – $h(x) = \frac{1}{x-1}$, finding which statement is <u>not</u> true.

This can be entirely done on the calculator.

$$h: (-1,1) \to R, h(x) = \frac{1}{x-1}$$

$$Define\ h(x) = \frac{1}{x-1}$$

A. $h(x)h(-x) = -h(x^2)$ **true**

B. $h(x) + h(-x) = 2h(x^2)$ **true**

C. $h(x) - h(0) = xh(x)$ **true**

D. $h(x) - h(-x) = 2xh(x^2)$ **true**

E. $\left(h(x)\right)^2 = h(x^2)$

For **E.**, we will get,

$$\frac{1}{(x-1)^2} = \frac{1}{x^2-1},$$

And there are no solutions between $-1 < x < 1$.

$$\left(h(x)\right)^2 \neq h(x^2)$$

The answer is E. $\left(h(x)\right)^2 \neq h(x^2)$ **ANSWER**

Question 14 – The variance of X.

Click here for **Maths**Methods.com.au resource: *Discrete Random Variables*

x	-1	0	1
$\Pr(X = x)$	p	$2p$	$1 - 3p$

$\mathbf{Var}(\mathbf{X}) = \mathbf{E}(\mathbf{X}^2) - [\mathbf{E}(\mathbf{X})]^2$

$\mathbf{E}(\mathbf{X})$ = Add up all $\mathbf{X} \times \mathbf{Pr}(\mathbf{X}) = -1 \times p + 0 \times 2p + 1 \times (1 - 3p) = 1 - 4p$

$\mathbf{E}(\mathbf{X}) = 1 - 4p$

$\mathbf{E}(\mathbf{X}^2) = (-1)^2 \times p + 0^2 \times 2p + 1^2 \times (1 - 3p) = 1 - 2p$

We use the calculator to simplify all of these.

$\mathbf{Var}(\mathbf{X}) = \mathbf{E}(\mathbf{X}^2) - [\mathbf{E}(\mathbf{X})]^2 = (1 - 2p) - (1 - 4p)^2 = \mathbf{6}\boldsymbol{p} - \mathbf{16}\boldsymbol{p}^2$

The answer is D. $6p - 16p^2$ ANSWER

Question 15 – The maximum area of the rectangle on $y = -x^3 + 8$

Click here for **Maths**Methods.com.au resource: *Stationary Points*

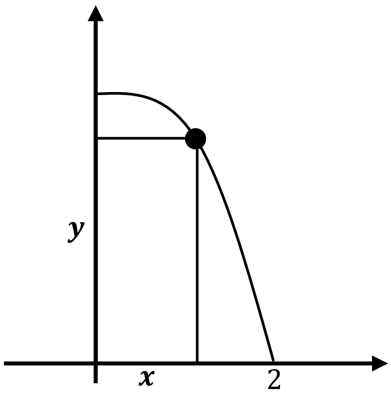
$\text{Area} = \text{width} \times \text{height}$

$\text{Area} = x \times y$

$\text{Area} = x \times (-x^3 + 8) = -x^4 + 8x$

$fmax(-x^4 + 8x, x)|0 < x < 2$

The question states that the triangle would only exist in this quarter, which could be a domain of 0 to 2 (see the graph)



$fmax(-x^4 + 8x, x)|0 < x < 2 \quad \rightarrow \quad x = 2^{\frac{1}{3}} = \sqrt[3]{2}$

Sub this back into Area

$\text{Area} = -x^4 + 8x = -(\sqrt[3]{2})^4 + 8(\sqrt[3]{2}) = \mathbf{6\sqrt[3]{2}}$

The answer is B. $\text{Area} = 6\sqrt[3]{2}$ ANSWER

Question 16 – Sample size of 5. $\Pr(\hat{p} = 0) = \frac{1}{243}$, then $\Pr(\hat{p} > 0.6) = ?$

Click here for **Maths**Methods.com.au resource: *Statistics*

$\hat{p}$ means sample proportion, $\hat{p} = \frac{x}{n}$

x means number in sample with the attribute

n means the total size of the sample, $n = 5$

$\Pr(\hat{p} = 0) = \frac{1}{243}$ $\hat{p} = \frac{x}{n}$, so if $\hat{p} = 0$ then $x = 0$ $\Pr(X = 0) = \frac{1}{243}$

Large population (Australians) means we can use binomial distribution to find p

$$\Pr(X = x) = \binom{n}{x} p^x (1 - p)^{n-x}$$

$n = 5$ $x = 3$ $\Pr(X = 0) = \frac{1}{243}$

$$\Pr(X = 0) = \binom{5}{0} p^0 (1 - p)^5 = (1 - p)^5 = \frac{1}{243}$$

$$solve\left((1 - p)^5 = \frac{1}{243}, x\right) \rightarrow p = \frac{2}{3}$$

$\Pr(\hat{p} > 0.6) = \Pr\left(\hat{p} > \frac{3}{5}\right)$, hence $x = 3$

$$\Pr(X > 3) = \Pr(X \geq 4) = \Pr(X = 4) + \Pr(X = 5)$$

Using calculator, **BinomialCDF** means adding a bunch of binomial distributions together

$$binomCdf\left(5, \frac{2}{3}, 4, 5\right) = 0.4609$$

This could also be done by hand,

$$\Pr(X > 3) = \binom{5}{4}\left(\frac{2}{3}\right)^4\left(\frac{1}{3}\right)^1 + \binom{5}{5}\left(\frac{2}{3}\right)^5\left(\frac{1}{3}\right)^0$$

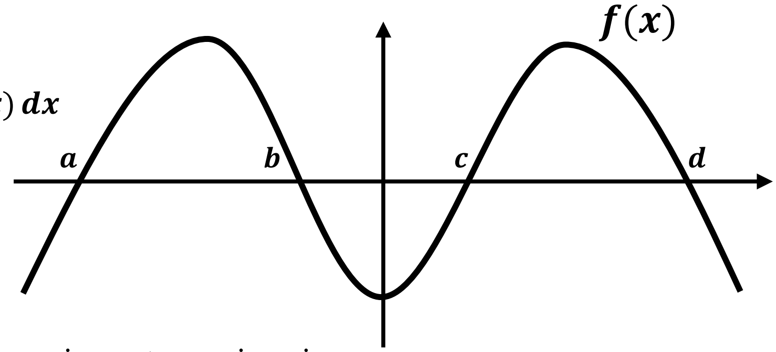
The answer is C. $\Pr(X > 3) = 0.4609$ ANSWER

Question 17 –The area bound by the curve and x-axis

Click here for **Maths**Methods.com.au resource: *Basics of Integration*

Area $= \int_a^b f(x)\,dx - \int_b^c f(x)\,dx + \int_c^d f(x)\,dx$

But this is not one of the answers.



Note that $f(-x) = f(x)$, in other words the y-axis creates a mirror image.

A. $\int_a^d f(x)\,dx$ is signed area, therefore would minus the area below the x-axis from the total, which is not what we want.

B. $\int_a^b f(x)\,dx - \int_c^b f(x)\,dx + \int_c^d f(x)\,dx$, c and b are reversed, this means that the value of the integral becomes positive.

C. $2\int_a^b f(x)\,dx + \int_b^c f(x)\,dx$,

$2\int_a^b f(x)\,dx$ the area between a and b is the same as between c and d.

$\int_b^c f(x)\,dx$ is a negative value so would need to be subtracted.

D. $2\int_a^b f(x)\,dx - 2\int_b^{b+c} f(x)\,dx$,

$b + c = 0$ b is the negative value of c

$2\int_b^0 f(x)\,dx$, would give the area from b to c.

This works!

E. $\int_a^b f(x)\,dx + \int_c^b f(x)\,dx + \int_d^c f(x)\,dx$

$\int_d^c f(x)\,dx$ would create a negative area.

The answer is D. $2\int_a^b f(x)\,dx - 2\int_b^{b+c} f(x)\,dx$ ANSWER

Question 18 –Find the smallest number of trails for $p \leq 0.01$

Binomial means that $E(X) = np$ and $SD(X) = \sqrt{np(1-p)}$

Where n is number of trails and p is probability.

In this case,

$E(X) = SD(X)$ which means $np = \sqrt{np(1-p)}$

$solve\left(np = \sqrt{np(1-p)}, p\right) \rightarrow p = 0 \quad p = \frac{1}{n+1}$ $\left(\text{which is same as } \frac{n}{n^2+n}\right)$

$p \neq 0$ because in the question it states that $0 < p < 1$

$p \leq 0.01$ and $p = \frac{1}{n+1}$

$\frac{1}{n+1} \leq 0.01$

$solve\left(\frac{1}{n+1} \leq 0.01, n\right) \rightarrow \boldsymbol{n < -1} \quad or \quad \boldsymbol{n \geq 99}$

The answer is D. $n \geq 99$ ANSWER

Question 19 – The value of *k* in the probability density function

Click here for **Maths**Methods.com.au resource: *Continuous Random Variables*

$$f(t) = \begin{cases} \cos(x) + 1 & k < x < (k+1) \\ 0 & \text{elsewhere} \end{cases}$$

Where $0 < k < 2$

A probability density function has an area of 1.

$$\int_k^{k+1} \cos(x) + 1\, dx = 1$$

To find k,

$$solve\left(\int_k^{k+1} \cos(x) + 1\, dx = 1, k\right) | 0 < k < 2$$

$k = \dfrac{\pi - 1}{2}$ (which is the same as $k = \dfrac{\pi}{2} + \dfrac{1}{2}$)

The answer is D. $k = \frac{\pi-1}{2}$ **ANSWER**

Question 20 – The ratio of area under graphs to area of triangle

Click here for MathsMethods.com.au resource: Basics of Integration

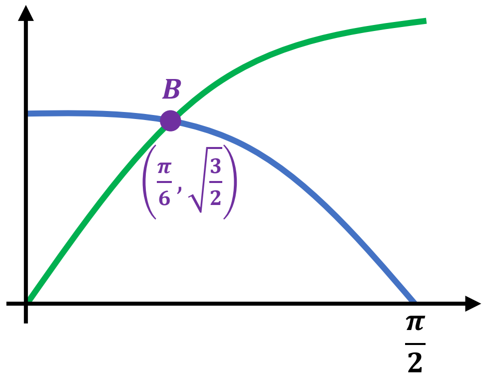
The x-value of B is at the intersection of the graphs

$solve(\sqrt{3}\sin(x) = \cos(x), x) | 0 < x < \frac{\pi}{2}$ $\quad x = \frac{\pi}{6}$

The y-value is found by subbing this into either equation,

$$\cos\left(\frac{\pi}{6}\right) = \sqrt{\frac{3}{2}}$$

Coordinates of the intersection **B is** $\left(\frac{\pi}{6}, \sqrt{\frac{3}{2}}\right)$



Area under graphs $= \int_0^{\frac{\pi}{6}} \sqrt{3}\sin(x)\,dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(x)\,dx = \frac{3}{2} - \frac{\sqrt{3}}{2} = \sqrt{3} - 1$

Area under triangle $= \frac{1}{2} \times \text{base} \times \text{height} = \frac{1}{2} \times \frac{\pi}{2} \times \sqrt{\frac{3}{2}} = \frac{\pi\sqrt{3}}{8}$

Height of triangle occurs at the intersection where $x = \frac{\pi}{6}$, which can be subbed into either equation.

Area under graphs: Area under triangle $= \sqrt{3} - 1 : \frac{\pi\sqrt{3}}{8}$

The answer is B. $\quad \sqrt{3} - 1 : \frac{\pi\sqrt{3}}{8}$ **ANSWER**

Done! That's all of multiple choice for 2017!

Got any feedback? I wanna hear it all! Email me at:

alex@mathsmethods.com.au

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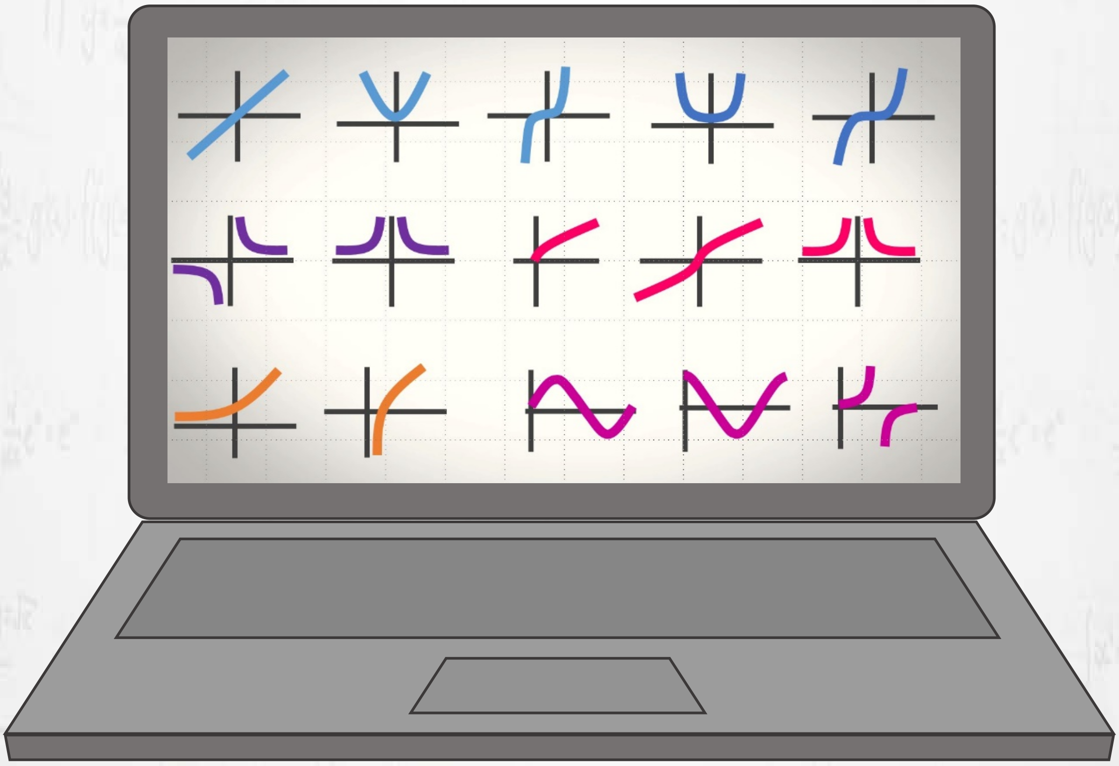

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